

PAPER 28 – Mathematical Model for the Prediction of the Transport and Fate of Oil Spill

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ABSTRACT

The mathematical model (MT) was used to identify and predict oil spill (OS) movement. The MT was developed based on the solution governing the partial differential equations of flow to simulate the processes of spreading and advection of oil slick on the surface and the oil droplet concentration distribution in the H₂O column, respectively. The oil movement (OM) was estimated by the vector sum of the wind drift, the surface current, spreading and diffusion. The transport of the oil phases in the water (H₂O) column was computed using the advection–diffusion equation. The variables determined were: oil spread rate on the H₂O surface, concentration of oil per area of spread in the H₂O column, concentration of oil per time in the H₂O column and spreading area per time. The kinetic models were developed to determine the rate of OM on the H₂O surface and column. The result obtained indicated that the concentration of the OS on H₂O surface per surface area of spread decreases as the surface area increases while the concentration of the OS decreases with time. The concentration of the OS per unit area at an area of 100m² reduced from 0.06432 to 0.01114 kg/ m³ at 200 m².

KEYWORDS: Oil spill, Wind speed, Oil slick, Time, Water column

1. INTRODUCTION

Rapid economic growth has caused a significant increase in fossil fuel consumption in recent decades. World oil production increased to about 74 million barrels per day by January 2005, and was fairly constant until 2011 when it started to increase to 77.8 mb/d in 2014 (Murray, 2015). The world production of crude oil is about 3 billion tons per year and half of it is transported by sea (Goldemberg, 2006; Ibeawuchi, 2016). Oil is often spilled during its transportation through the sea in accidental discharge or through accident during road transportation with the aid of tankers. Most oil has a density less than water, which makes it to float in water. Oil tends to spread into a thin layer on the surface of water as sheen (Claireaux and Davoodi, 2010).

The discharged oil is of different varieties such as [crude oil](#), refined petroleum products (such as [gasoline](#) or [diesel fuel](#)) or by-products and can be further categorized into light, medium and heavy oils. Very light oils such as jet fuel, and gasoline is highly volatile and evaporate quickly. Very light oils are one of the most acutely toxic oils and generally affect aquatic life (fish, invertebrates, and plants) that live in the upper water column. Light oils (diesel, light crude, heating oils) are moderately volatile and can leave a residue of up to one-third of the amount spilled after several days. Light oils leave a film on intertidal resources and have the potential to cause long-term contamination. Medium oils (most crude oils) and heavy oil can cause severe and long-term contamination to intertidal areas but medium oils are less likely to mix with water, while heavy oils (heavy crude, No. 6 fuel oil and Bunker C) do not readily mix with water and have far less evaporation and dilution potential (Okoye and Okunrobo, 2014). Heavy oils have severe impacts on waterfowl and fur-bearing marine mammals. These oils tend to weather slowly. Clean-up of heavy oil is difficult and usually a long-term process. Very heavy oils can float, mix, sink, or hang in the water. These oils can become oil drops and mix in the water, accumulate at the bottom,

mix with sediment and then sink. (Anderson et al., 1993). When liquid oil is spilled on the sea surface, it spreads over the surface of the water and forms a thin film called, oil slick (Okoye and Okunrobo, 2014). The movement of the slick is governed by the advection and turbulent diffusion due to current and wind action (Koch and Bavumiragira, 2019).

When an oil spill occurs on water, the three (3) major step of controlling and managing it is as follows; mechanical containment, recovery and clean up. When an oil spill happens on water, it is critical to contain the spill as quickly as possible to minimize danger and potential damage to persons, property, and natural resources. Containment equipment is used to restrict the spread of oil and to allow for its recovery, removal, or dispersal. The most common types of equipment used to control the spread of oil and recover oil from the surface are floating barriers, called booms, skimmers sorbents, netting systems, sorbent booms, improvised booms and barriers, bubble barriers and chemical barriers, while the clean-up techniques are bioremediation and dispersants. Bioremediation involves the use of [microorganisms](#) or [biological agents](#) to break down or remove oil. Selection of the most appropriate for a specific oil spill situation will depend on the particular conditions. Once an oil spill has been contained, efforts to remove the oil from the water can begin. After a large oil spill, the oil slick is sometimes treated with a dispersant. Dispersants emulsify petroleum by reducing the interfacial tension between petroleum and water. The small droplets that are formed are dispersed into a water column to a depth of several meters, preventing wind-induced drift of the oil slick. It is claimed that treatment by a dispersant enhances the biodegradation of petroleum. Oil spillage affects both animals and human staying close to the shore or those whose livelihood depends on the surrounding water and ultimately trigger health-associated problems. For effective management of oil spread on the surface of the water, laboratory investigations have been conducted to study the variables that affect the spread of oil with some submissions that makes it possible to manage it. Extending laboratory work by using them as a basis for the prediction of oil spills will assist in monitoring and taking proactive decisions to minimize the effect of spillage in the environment.

2. METHODOLOGY

2.1 Model Development for Spreading and Advection of oil spills

In the majority of cases, a mathematical model is the only available tool for rapid computation of the spilled oil fate, and for simulation of the various clean-up operations. Two major processes transport oil spilled on water: spreading and advection. For small spills (<100 barrels), the spreading process is complete within the first hour of the release. Winds, currents, and large-scale turbulence (mixing) are advection mechanisms that can transport oil at great distances.

In general, the oil movement can be estimated as the vector sum of the wind drift (using 3% of the wind speed), the surface current, spreading and larger-scale turbulence (diffusion).

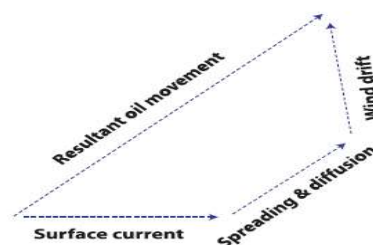


Fig 2: Oil Movement

The governing equation for oil spread on the water in radial coordinate (r), is derived by taking an elemental volume of oil.

(Rate of mass input) – (Rate of mass Output) + (Rate of production from Source) = (Rate of production from source)

2.1.1 Convective Movement

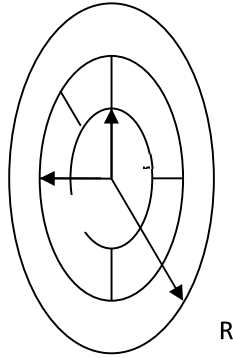


Fig 3: Convective Radial Movement

Rate of Mass Input of oil spill = *velocity* × *area* × *concentration*

The velocity of water current in the radial direction = U_{A0} ; Area of circle (at r) = $4\pi r^2 \big|_r$; Concentration (at r) = $C_{A0} \big|_r$

$$\therefore \text{Rate of Mass Input} = U_{A0} \cdot 4\pi r^2 C_{A0} \big|_r \quad (1)$$

Rate of mass output of the spill = *velocity* × *area* × *concentration* ; Velocity of water current in radial direction = U_{A1} ; Area of circle (at r+dr) = $4\pi r^2 \big|_{r+dr}$; Concentration (at r+dr) = $C_{A1} \big|_{r+dr}$;

$$\therefore \text{Rate of Mass Output} = U_{A1} \cdot 4\pi r^2 C_{A1} \big|_{r+dr} \quad (2)$$

2.1.2 Diffusive Movement

Rate of Mass Input = area × flux; Area of circle = $4\pi r^2$; Flux = $N_A = -D \frac{\partial c}{\partial r}$

$$\text{Rate of mass input} = -D \frac{\partial c}{\partial r} \big|_r \cdot 4\pi r^2 \big|_r = -4\pi r^2 D \frac{\partial c}{\partial r} \big|_r \quad (3)$$

$$\text{Rate of mass output} = -D \frac{\partial c}{\partial r} \big|_{r+dr} \cdot 4\pi r^2 \big|_{r+dr} = -4\pi r^2 D \frac{\partial c}{\partial r} \big|_{r+dr} \quad (4)$$

Rate of Mass Input for Convection and Diffusion

$$\text{Rate of mass input} = U_{A0} \cdot 4\pi r^2 C_{A0} \big|_r - 4\pi r^2 D \frac{\partial c}{\partial r} \big|_r \quad (5)$$

Rate of Mass Output for Convection and Diffusion

$$\text{Rate of mass output} = U_{A1} \cdot 4\pi r^2 C_{A1} \big|_{r+dr} - 4\pi r^2 D \frac{\partial c}{\partial r} \big|_{r+dr} \quad (6)$$

Rate of Production from source

$$\text{Rate of production} = R_A \times 4\pi r^2 dr \quad ; \text{Source term} = R_A \quad (7)$$

Rate of Accumulation

$$\text{Rate of Accumulation} = \text{Volume} \times \text{rate of concentration} = 4\pi r^2 dr \times \frac{\partial c}{\partial t} \quad (8)$$

Combining equations 5, 6, 7, 8

$$\left[U_{A0} \cdot 4\pi r^2 C_{A0} \Big|_r - 4\pi r^2 D \frac{\partial c}{\partial r} \Big|_r \right] - \left[U_{A1} \cdot 4\pi r^2 C_{A1} \Big|_{r+dr} - 4\pi r^2 D \frac{\partial c}{\partial r} \Big|_{r+dr} \right] + R_A \times 4\pi r^2 dr = 4\pi r^2 dr \times \frac{\partial c}{\partial t} \quad (9)$$

$$\frac{\left[U_{A0} C_{A0} \Big|_r - D \frac{\partial c}{\partial r} \Big|_r \right]}{\partial r} - \frac{\left[U_{A1} C_{A1} \Big|_{r+dr} - D \frac{\partial c}{\partial r} \Big|_{r+dr} \right]}{\partial r} + R_A = \frac{\partial c}{\partial t} \quad (10)$$

$\lim dr \rightarrow 0$

$$\frac{\partial c}{\partial t} = \frac{d}{dr} \left[U_A C_A - D \frac{\partial c}{\partial r} \right] + R_A \quad (11)$$

Similarly;

$$\frac{\partial h}{\partial t} = \frac{d}{dr} \left[U h_A - D_A \frac{\partial h}{\partial r} \right] + R \quad (12)$$

$$D_A = g h^2 (\rho - \rho_0) \frac{\rho_0}{\rho f} \quad (13)$$

Boundary Conditions

$$\frac{\partial C_A}{\partial r} = 0 \quad \text{at } r=0 \quad \text{at the centre} \quad ; \quad \frac{\partial C_A}{\partial r} = 0 \quad \text{at } r=R$$

2.2 Numerical Solution

2.2.1 Computational Techniques

Using the Governing Equation 11

$$\frac{\partial c}{\partial t} = \frac{d}{dr} \left[U_A C_A - D \frac{\partial c}{\partial r} \right] + R_A \quad ; \quad \frac{\partial c}{\partial t} = U_A \frac{\partial C_A}{\partial r} - D \frac{\partial^2 c}{\partial r^2} + R_A \quad (14)$$

In the presence of strong currents, the mass transport phenomenon becomes advection-dominant, which increases the requirement for the accuracy of a numerical scheme to be applied for the integration of the transports Equation 14. To decrease the numerical diffusion (approximation error) below the physical one, the accuracy of approximation of the advection terms. Development of a high-order numerical scheme for advection terms can be illustrated as follows:

Assuming the diffusion term is Zero

$R_A = 0$ (Since the source is instantaneous)

$$\frac{\partial c}{\partial t} = U_A \frac{\partial C_A}{\partial r} \quad (15)$$

2.2.2 Explicit Method of Finite Difference

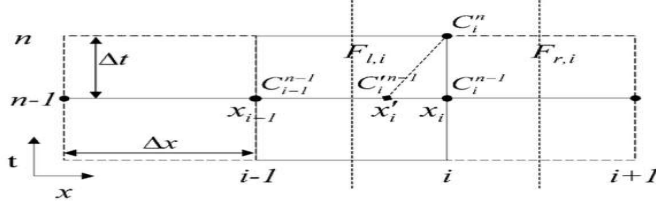


Fig 4: Subdivision of the r-t domain into intervals of Δr and Δt for finite-difference representation of the one-dimensional, time-dependent fluid advection equation.

Then, the concentration $C[r, t]$ at a location r and a time t is denoted by the symbol C_i^n

$$C[r, t] = C[i\Delta r, n\Delta t] = C_i^n \quad (16)$$

Various derivatives appearing in the above fluid convection equation are approximated as above.

The first derivative of concentration with respect to the time variable (t) at a position ($i\Delta r$) and at a time ($n\Delta t$) is represented by;

$$\frac{\partial c}{\partial t} \Big|_{i,n} = \frac{C_i^{n+1} - C_i^n}{\Delta t} \quad (17)$$

The first derivative of concentration with respect to the radius (r) at a position ($n\Delta x$) and at a time ($i\Delta t$) is represented by;

$$\frac{\partial C_A}{\partial r} \Big|_{i,n} = \frac{C_n^{i+1} - C_n^i}{\Delta r} \quad (18)$$

Substituting equations 3.18 and 3.17 into equation 3.15

The finite difference form of the time-dependent fluid-convection equations becomes:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = U_A \frac{C_n^{i+1} - C_n^i}{\Delta r} \quad (19)$$

$$C_i^{n+1} - C_i^n = U_A \frac{\Delta t}{\Delta r} C_n^{i+1} - C_n^i ; \quad \therefore \alpha = U_A \frac{\Delta t}{\Delta r} = \text{Courant Number} \quad (20)$$

$$C_i^{n+1} - C_i^n = \alpha [C_n^{i+1} - C_n^i] ; \quad C_i^{n+1} = C_i^n + \alpha [C_n^{i+1} - C_n^i] \quad (21)$$

2.3 Computational Technique

2.3.1 The oil spreading Law

Gravity, inertia, viscosity and surface tension forces cause the horizontal expansion of an oil slick (i.e. spreading).

Oil spreading Radius (OSR) = Gravity inertia + Viscosity + Surface tension force. Using experimental evidence, Lehr *et al.*(2003) suggested that an oil slick spreads as an ellipse according to the following:

$$A_e = \frac{\pi}{4} \times R_{\max} \times R_{\min} \quad (22)$$

$$R_{\min} = 1.7 \left[\frac{\rho - \rho_0}{\rho} \right]^{1/3} V^{1/3} t^{1/4} \quad (23)$$

$$R_{\max} = R_{\min} + 0.03 U^{4/3} t^{3/4} \quad (24)$$

Using the modified Fay-type spreading equation considering the influence of wind

$$A = 2290 \left[\frac{\Delta\rho}{\rho_0} \right]^{2/3} V^{2/3} + 40 \left[\frac{\Delta\rho}{\rho_0} V U_w \right]^{2/3} t \quad ; \Delta\rho = \rho - \rho_0 \quad (25)$$

Simulation Data (Lehr *et al.*, 2003)

The following data were used to simulate the model: $V = 600m^3$, $U_w = 10m/s, 20m/s$, $\rho = 1000kg/m^3$, $\rho_0 = 840kg/m^3$, $E = 2m/s^2$, $u = 0.02m/s$, $Z_m = 1.5m$, $f = 0.02kg/m^3s$ and $C_{\min} = 1 \times 10^{-8}$

3. RESULTS AND DISCUSSION

3.1 Simulation of area of spreading with time using Matlab

It was observed from the graph (Figure 5&6) that rate of spread of the oil spill increases with time covering more areas. The rate of spreading was greatly influenced by the meteorological condition of the water body. At a wind speed of 10 m/s, the oil spread to an area of 40 m² within 300 seconds. At a wind speed of 20 m/s, the rate of spreading is almost double.

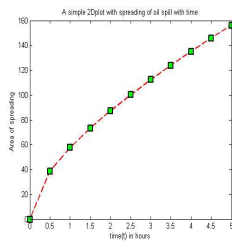


Fig 5: Plot of Area of spill against time

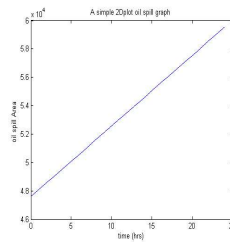


Fig 6: 2D Plot of oil spill Area against Time

3.2 Simulation of volumetric concentration of oil per area of spread

The graph (Figure 7) shows the volumetric concentration of oil per area of spread. it can be deduced that the concentration of the oil spill on the water surface per surface area of spread decreases as the area increases. The concentration of the spilled oil per unit area at an area of 100m² reduced from 0.06432 to 0.01114 kg/ m³ at 200 m². This is due to the effect of other weathering processes like evaporation, emulsification, dispersion and other oil fate processes that occur when the oil enters the water column.

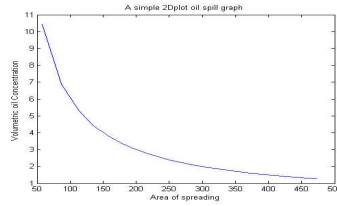


Fig 7: 2D Plot of Volumetric oil concentration against the Area of Spreading

3.3 Simulation of volumetric concentration of oil spill with time

From Figures 8a and b, it was observed that the concentration of oil spills decreases with time. This is due to the fact that as the oil spread with time, the surface area of the oil increases thus leading to faster evaporation of the light oil fraction and the entrainment of the rest component into the water column. Comparing the result of the simulated data of the oil spill, it was observed that the modified oil spreading model gave a better simulation of the spill situation with the inclusion of the wind speed.

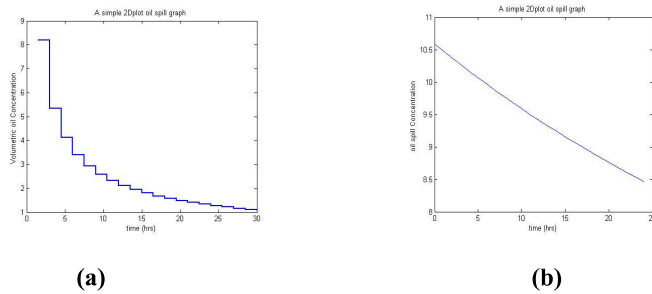


Fig 8: 2D Plot of Volumetric oil (a) and oil spill concentration (b) against time

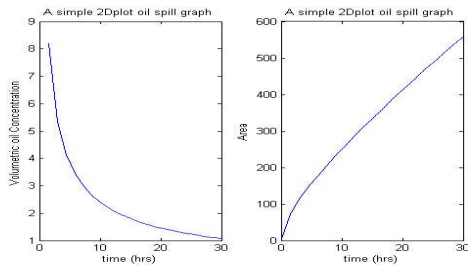


Fig 9: 2D Plot of Volumetric oil concentration and Area of Spreading with time

4. CONCLUSION

An oil spreading model has been developed to predict the movement of surface oil slicks on water bodies. Using this model, the area of spreading and concentration of the oil spill at different time intervals can be solved. With the required simulation data input into the model, the simulation results show the process of oil slick movement on the water surface. This model would help to develop proper contingency plans as well as assist in deciding the best containment and recovery approach in case of a spill. Though the model can predict the concentration of oil spill and the area of spread at different time intervals, the followings are recommended:

- The meteorological data of water bodies should be readily available for immediate simulation in case of an oil spill
- Different oil spill models should be tested to discover the best that would simulate the accurate condition of the oil spill

- Environmental consequences of the combating techniques application have to be carefully calculated before the actual usage.

REFERENCES

- Anderson, E.L., E. Howlett, K. Jayko, V. Kolluru, M. Reed, and Spaulding, M. (1993). The worldwide oil spill model (WOSM): an overview in Proceedings of the 16th Arctic and Marine Oil Spill Program, Technical Seminar. Ottawa, Ontario: Environment Canada. 627–646.
- Bird, R. B., Stewart, E. W. Warren, Lightfoot, N. E. (2006). Transport Phenomena, Wiley, India. p.78
- Kuch, G.S. and Bavumiragira, J.P. (2019). Impacts of crude oil exploration and production on environment and its implications on human health: South Sudan Review. International Journal of Scientific and Research Publications, 9 (4), 247-256.
- Lehr and Smith (2003). The spread of oil slicks on a calm sea, in oil on the sea. New York: McGraw-Hill, p.110-120.
- NOAA (2002). Environmental Sensitivity Index Guidelines, version 3.0. NOAA Technical Memorandum NOS OR&R 11. Seattle: Hazardous Response and Assessment Division, National Oceanic and Atmospheric Administration, 129p.
- Nwilo and Badejo (2005). Causes of oil spills in Nigeria, African News 13: 14.
- Okoye, C.O. and Okunrobo, L.A. (2014). Impact of oil spill on land and water and its health implications in Odu-aboro community, Sagamu, Ogun State, Nigeria. Available at http://www.scenrp.com/World%20Journal%20of%20Environmental%20Sciences%20&%20Engineering/WJESE_Vol.%201,%20No.%201,%20January%202014/IMPACT%20OF%20OIL.pdf> assessed on 17th June, 2016.
- Prince R., (1993).Petroleum Spill Bioremediation in Marine Environments. Critical Rev. Micobiol. 19(4): 217-242.
- Rossouw, M., (1998). Oil Spill Simulation: Reducing the Impact. START/IOC/LOICZ Workshop on Climate Change and Coastal Process in Cotonou, Benin, West
- Wikipedia, (2008). Oil spill incidents. <http://www.wikipedia.com>.