

PAPER 70 – H-Infinity Mixed Sensitivity Function for the DC Motor Control

A. Umar^{1*}, N.S Usman¹, S.M. Yusuf¹, Z. Haruna¹, A. Ore-Ofe¹, E.S. Jeffrey¹, S.A. Mikail²

¹Department of Computer Engineering, Ahmadu Bello University, Zaria.

²Department of Electronics & Telecommunications Engineering, Ahmadu Bello University, Zaria.

*Email: abubakaru061010@gmail.com

ABSTRACT

This article presents an H-infinity mixed sensitivity function for the DC motor control. The main purpose of the DC motor is controlling the speed on a precise reference when uncertainties and disturbances act on it. Regulating the power supply of the DC motor in order to control the angular speed is a challenging task, this necessitates the use of H-infinity mixed sensitivity function, which is a robust technique to control the DC motor's speed in the presence of uncertainties and disturbances by the selection of appropriate weights for the plant. The DC motor was modelled, and simulation was done in MATLAB 2022b environment. The results showed that for the sensitivity function of the DC motor, the bode magnitude plot reduces from 35 dB to -35 dB with a frequency of 0.01 rad/secs to 1000000 rad/secs. Also, for the complementary sensitivity function of the DC motor, the bode magnitude plot reduces from 19 dB to -33 dB with a frequency of 0.01 rad/secs to 1000000 rad/secs. Taking the singular value plot into consideration, the magnitude of the DC servo motor reduces from -33 dB to -77 dB which showed good robust stability, performance and tracking capability when using H-infinity mixed sensitivity function.

KEYWORDS: DC Motor, H-infinity, Mixed Sensitivity Function, Robust Stability, Robust performance

16. INTRODUCTION

The Direct current (DC) motor is a device that converts electrical power to mechanical power. There exists numerous variants of DC motors which are; brushless (BLDC) motor, brushed DC motor, stepper motor and servo motor (Ma'arif & Setiawan, 2021). DC motors find applications in areas like pendulum, line follower robot, maze following robot and robot balancing (Covaci *et al.*, 2020; Hsu *et al.*, 2018; Tayal *et al.*, 2020). It possesses some advantages over the AC motors which are ease of control characteristics, low power consumption and better performance (Pandey *et al.*, 2020). The main purpose of the DC motor is controlling the speed on a precise reference when uncertainties and disturbances act on it (Taut *et al.*, 2018). The angular speed is regulated by adjusting the power supply of the motor, which is a challenging task, hence the need for a controller. However, the DC motor's speed robustness ensures that the system functions excellently in the presence of disturbance and uncertainty, while motors accuracy is being affected by its disturbance (Ahmed *et al.*, 2018). The accuracy and stability of the DC motor relates to the disturbance and uncertainty of the plant. There are some features like speed and torque nonlinearities which produces uncertainties in the rotational speed control system (Prakosa *et al.*, 2021b). The main disturbance occurs due to environmental circumstances and overload (Krishnan *et al.*, 2017). The position sensor's sensitivity causes uncertainty mainly in noise situations (Prakosa *et al.*, 2020). However, robust control provides a controller design which gives a tradeoff with uncertainty (Prakosa & Vtorov, 2019). H-infinity controller, which is a branch of robust controller gives a numerical optimization problem of stabilizing a plant in the presence of disturbance and uncertainties (Prakosa & Stotckaia, 2019). The H-infinity mixed sensitivity function is used for shaping the plant, in which the corresponding weights penalizes either the error signal, the input signal or the output signal through the use of augmented plant and weighting filters (Prakosa *et al.*, 2021a; Umar *et al.*, 2018).

There are some literatures done within the field of DC motor with respect to robust control technique. Dey *et al.* (2016) developed a robust H-infinity controller for the DC motor. The result was compared with the conventional PID controller, which performed better than the PID controller in terms of robust stability in the time and frequency domain. Also, (Vinida & Chacko, 2016) presented an H-infinity controller for the control of a sensorless DC motor. Particle swarm optimization (PSO) algorithm was used to optimize the H-infinity controller. The controller was also compared with PID controller, which showed better performance than the PID controller in terms of good disturbance rejection and reference tracking. The weights of the H-infinity controller were optimized in (Vinida & Chacko, 2021) using PSO, in which the DC motor was controlled using H-infinity controller. The experimental results of the controller were compared with proportional-integral (PI) controller. The H-infinity controller performed better during transient and sudden disturbances in the load. The DC motor's robustness was improved in (Prakosa *et al.*, 2022) using H-infinity mixed sensitivity synthesis technique to solve the problem of uncertainties and disturbances. The robustness of the DC motor's speed was improved because the system had a better overshoot and smoother signal.

This article proposes an H-infinity mixed sensitivity function for the DC motor control. The rest of the paper is structured as follows; the introduction is discussed in the first section, while in the second section, the DC motor model is presented. In section three, H-infinity mixed sensitivity function is designed and section four presents the simulation results. Finally, the last section gives the conclusion.

17. DC MOTOR MODEL

The DC motor considered in this article is the self-excited type. The motor is excited with a constant field current and a constant flux. The speed of the motor is regulated by changing the armature voltage. Hence, the motor is an armature based controlled motor (Porbiya & Kulkarni, 2021). The DC motor model parameters is adopted from (Akbar *et al.*, 2016).

The DC motor schematic diagram is given in Figure 1.

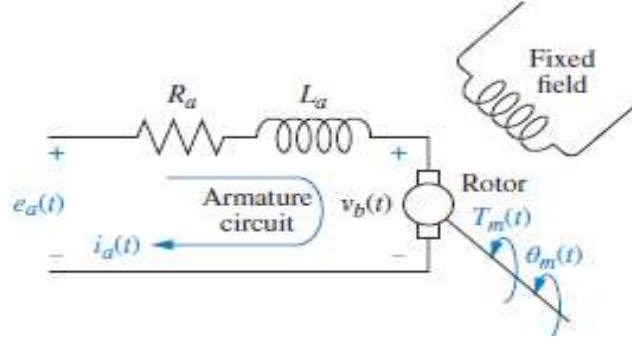


Figure.1. Schematic Diagram of the DC Motor (Nise, 2011)

The back electromotive force (emf) V_b is given by (Porbiya & Kulkarni, 2021):

$$V_b = K_b s \theta_m (s) \quad (1)$$

Where K_b is the back emf and θ_m is the maximum rotor shaft angle.

The armature voltage using Kirchoff's voltage law is written as (Porbiya & Kulkarni, 2021):

$$E_a (s) = R_a I_a + L_a s I_a (s) + V_b (s) \quad (2)$$

The developed motor torque is proportional to the armature current, which is written as (Porbiya & Kulkarni, 2021):

$$T_m (s) = K_t I_a (s) \quad (3)$$

Where T_m is the torque developed by the motor, K_t is the motor torque constant which depends on the magnetic field and motor characteristics.

Hence, the DC motor transfer function is written as (Porbiya & Kulkarni, 2021):

$$\frac{\theta_m (s)}{E_a (s)} = \frac{K_t / (R_a J_m)}{s \left[s + \frac{1}{J_m} \left(D_m + \frac{K_t K_b}{R_a} \right) \right]} \quad (4)$$

18. H-INFINITY CONTROLLER

The augmented plant model of the H-infinity controller is written as (Umar *et al.*, 2018):

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (5)$$

While the state space augmented model is written as (Umar, 2017):

$$\dot{x} = Ax + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (7)$$

The closed loop transfer function is given as (Umar, 2017):

$$T y_1 u_1 (s) = P_{11} (s) + P_{12} (s) [I - F(s) P_{22} (s)]^{-1} F(s) P_{21} (s) \quad (8)$$

The objective of robust control is to find a stabilizing controller.

$$u_2(s) = F(s)y_2(s) \quad (9)$$

Given the condition (Umar, 2017):

$$\|Ty_1u_1\|_\infty < 1 \quad (10)$$

The design objective is to find a stabilizing controller $F_c(s)$, which guarantees the closed system bounded by an H_∞ -norm that is given by a positive number

$$\|Ty_1u_1\|_\infty < \gamma \quad (11)$$

$$F_c(s) = \left[\begin{array}{c|c} A_f & -ZL \\ \hline F & 0 \end{array} \right] \quad (12)$$

The optimal criterion in designing an optimal H_∞ controller is written as (Umar, 2017):

$$\|Ty_1u_1\|_\infty < \frac{1}{\gamma} \quad (13)$$

3.1 H_∞ Mixed Sensitivity Problem

For the H_∞ optimal control, the weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$ were used for shaping the plant model $G(s)$. $W_1(s)$ penalizes the error signal, $W_2(s)$ penalizes the input signal and the output signal is been penalized by $W_3(s)$ [22]. The H_∞ mixed sensitivity problem control structure is given in Figure 2.

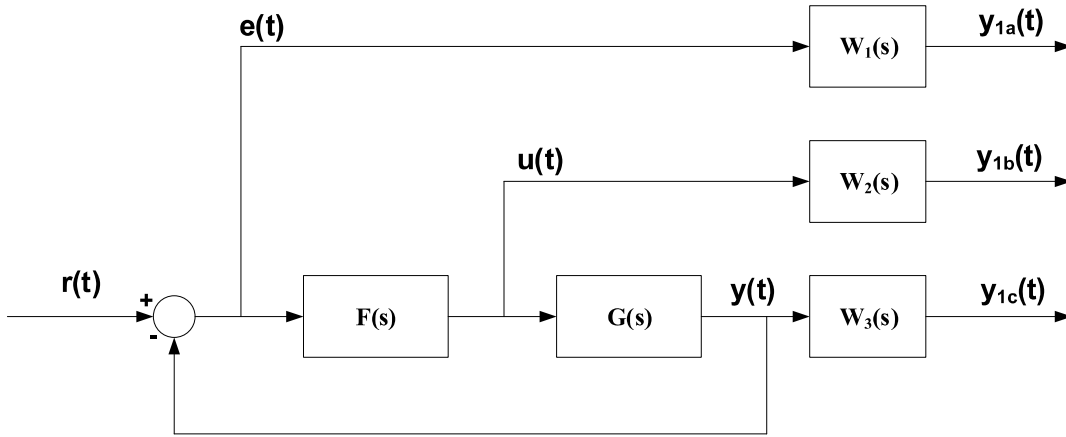


Figure 2: H_∞ Mixed Sensitivity Problem (Xue *et al.*, 2007)

The sensitivity transfer function $S(s)$ and complementary sensitivity function $T(s)$ are written as:

$$S(s) = [1 + F(s)G(s)]^{-1} \quad (14)$$

$$T(s) = 1 - S(s) = F(s)G(s)[1 + F(s)G(s)]^{-1} \quad (15)$$

3.2 H_∞ Controller Selection

The selected weighting functions are written as:

$$W_1(s) = \frac{0.1011s + 1.352}{s + 0.02317} \quad (16)$$

$$W_2(s) = 0.0243 \quad (17)$$

$$W_3(s) = \frac{s + 206.5}{0.02157s + 1872} \quad (18)$$

19. SIMULATED RESULTS

This section discusses the results obtained from the simulation, which was done using MATLAB 2022b environment. The DC motor transfer function is written as.

$$G(s) = \frac{148}{s^2 + 201s + 6290} \quad (18)$$

However, the system has been proved to be observable and controllable, which paved way for other control analysis. The augmented H-infinity controller plant model is given as:

$$\begin{bmatrix} A_g & B_g \\ C_g & D_g \end{bmatrix} = \begin{bmatrix} -201 & -6290 & 1 \\ 1 & 0 & 0 \\ 0 & 148 & 0 \end{bmatrix} \quad (19)$$

The sensitivity and inverse sensitivity function bode magnitude and phase plot is given in Figure 3.

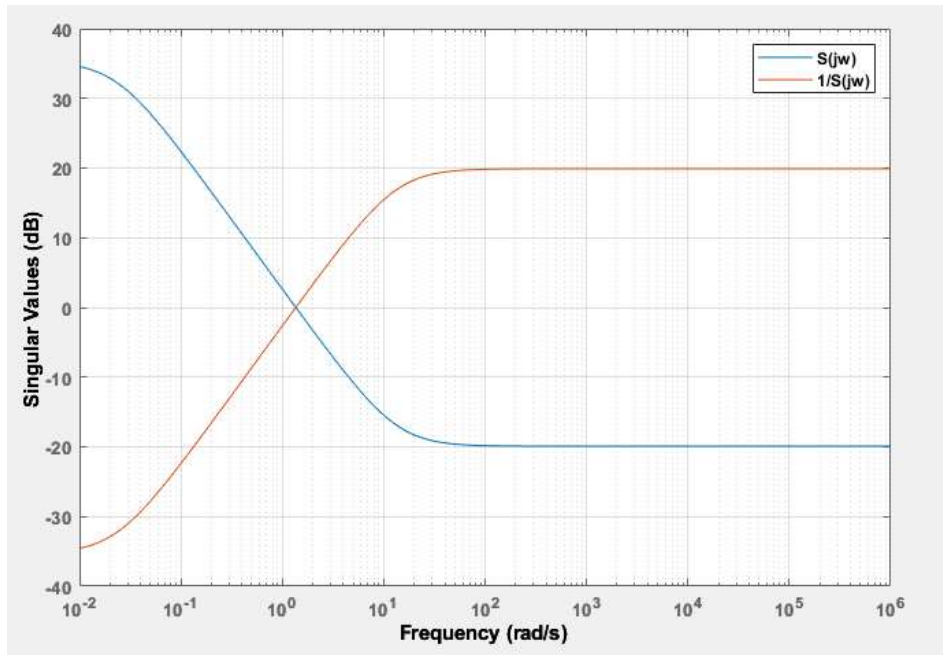


Figure 3: Sensitivity and Inverse Sensitivity Function Bode Plot

From Figure 3, it can be seen that as the frequency increases from 0.01 rad/sec to 10000000 rad/sec, the sensitivity function of the DC motor reduces from 35 dB to -35 dB. This can be deduced that at lower frequency, the sensitivity function is high, and at higher frequency, it becomes low. This shows the robustness of the H_∞ controller when applied to the DC servo motor.

The complementary and inverse complementary function bode magnitude and phase plot is shown in Figure 4.

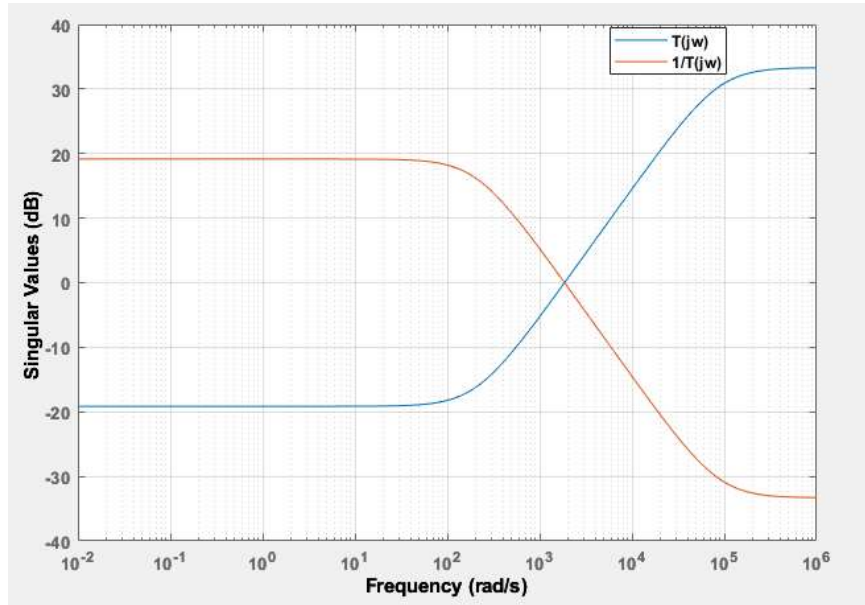


Fig. 4. Complementary and Inverse Complementary Function Bode Plot

From Figure 4, it can be seen that as the frequency increases from 0.01 rad/secs to 1000000 rad/secs, complementary function of the DC motor reduces 19 dB to -33 dB. This shows that at lower frequencies, the inverse complementary sensitivity function is high, and at low frequencies, the complementary sensitivity function is low. This shows the robustness of the H_∞ controller when applied to the DC servo motor.

Also, the singular value bode magnitude plot is shown in Figure 5.

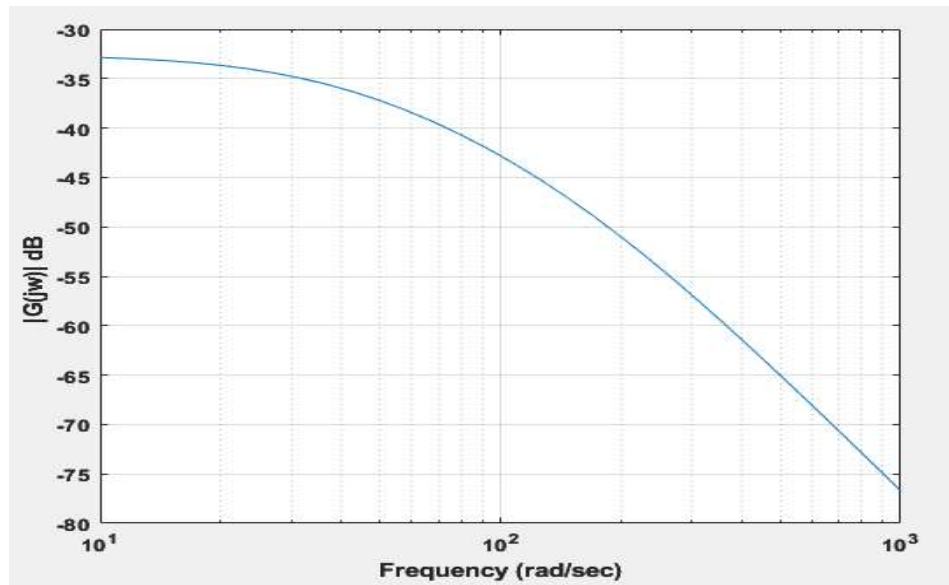


Figure 5. Singular Value Bode Magnitude Plot

From Figure 5, it can be seen that as the frequency increases from 10 rad/secs, the magnitude of the DC servo motor also reduces from -33 dB to -77 dB, which clearly shows the robustness of using H_∞ controller of the DC servo motor.

The designed H_∞ controller is given as:

$$K = \frac{1356.1479s^3 + 117968874.8478s^2 + 23665484281.8058s + 740309658505.3711}{s^4 + 87020.4342s^3 + 20254139.9253s^2 + 1108802337.4526s + 25680077.8285} \quad (20)$$

With a γ value of 0.96094, which is less than 1.

20. CONCLUSION

This paper proposes an H-infinity mixed sensitivity function for the DC motor control. The main purpose of the DC motor is controlling the speed on a precise reference when uncertainties and disturbances act on it. Regulating the power supply of the DC motor in order to control the angular speed is a challenging task, hence H-infinity mixed sensitivity function was used for the control of the DC motor's speed because it takes into consideration uncertainties and disturbances, by selection of appropriate weights for the plant. Simulation was carried out in MATLAB 2022b environment, and the results obtained showed that for the DC motor's sensitivity function, the bode magnitude plots reduces from 35 dB to -35 dB with a frequency of 0.01 rad/sec to 1000000 rad/sec. Also, for the complementary sensitivity function of the DC motor, the bode magnitude plots also reduces from 19 dB to -33 dB with a frequency of 0.01 rad/sec to 1000000 rad/sec. However, for the singular value bode plot, the magnitude of the DC servo motor reduces from -33 dB to -77 dB which shows good robust stability, performance and good tracking capability when using H_∞ mixed sensitivity function. Future research will consider applying H_∞ mixed sensitivity function on other variants of the DC motor, by comparing both simulated and experimental results.

REFERENCES

- Ahmed, A., Parmar, G., & Gupta, R. (2018). *Application of GWO in Design of Fractional Order PID Controller for Control of DC Motor and Robustness Analysis*. Paper presented at the 2018 International Conference on Advances in Computing, Communication Control and Networking (ICACCCN), Greater Noida, India.
- Akbar, M. A., Naniwa, T., & Taniai, Y. (2016). *Model reference adaptive control for DC motor based on Simulink*. Paper presented at the 2016 6th International Annual Engineering Seminar (InAES), Yogyakarta, Indonesia.
- Covaci, R., Harja, G., & Nascu, I. (2020). *Autonomous Maze Solving Robot*. Paper presented at the 2020 IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR), Cluj-Napoca, Romania.
- Dey, N., Mondal, U., & Mondal, D. (2016). *Design of a H-infinity Robust Controller for a DC Servo Motor System*. Paper presented at the 2016 International Conference on Intelligent Control Power and Instrumentation (ICICPI), PUNE, India.
- Hsu, C.-F., Su, C.-T., Kao, W.-F., & Lee, B.-K. (2018). *Vision-Based Line-Following Control of a Two-Wheel Self-Balancing Robot*. Paper presented at the 2018 International Conference on Machine Learning and Cybernetics (ICMLC), Chengdu, China.
- Krishnan, T. D., Krishnan, C., & Vittal, K. P. (2017). *Design of Robust H-infinity Speed Controller for High Performance BLDC Servo Drive*. Paper presented at the 2017 International Conference on Smart grids, Power and Advanced Control Engineering (ICSPACE), Bangalore, India.
- Ma'arif, A., & Setiawan, N. R. (2021). Control of DC Motor Using Integral State Feedback and Comparison with PID: Simulation and Arduino Implementation. *Journal of Robotics Control*, 2(5), 456-461. doi:10.18196/jrc.25122
- Nise, N. (2011). *Control System Engineering* (Vol. 7). Courier Kendallville: John Wiley & Sons, Inc, New York.
- Pandey, S. K., Bera, C., & Dwivedi, S. S. (2020). *Design of robust PID controller for DC Motor using TLBO algorithm*. Paper presented at the 2020 IEEE International Conference on Advances and Developments in Electrical and Electronics Engineering (ICADEE), Coimbatore, India.
- Porbiya, P., & Kulkarni, S. P. (2021). *Speed Control of DC Motor using MRAC, PSO and PSO Fuzzy Controller*. Paper presented at the 2021 International Conference on Smart Generation Computing, Communication and Networking (SMART GENCON), Pune, India.
- Prakosa, J., & Stotckaia, A. (2019). The h-infinity robust control for optimization on low water flow application. *Journal of Advanced Research in Dynamical Control Systems*, 11(S4), 1995-2006.
- Prakosa, J., & Vtorov, V. (2019). Experimental studies of adaptive control to stabilize the automatic unmanned mini quad rotor helicopter position. *Journal of Advanced Research in Dynamical Control Systems*, 11(S4), 1983-1994.
- Prakosa, J. A., Gusrialdi, A., Kurniawan, E., Stotckaia, A. D., Adinanta, H. J. I. J. o. D., & Control. (2022). Experimentally robustness improvement of DC motor speed control optimization by H-infinity of mixed-sensitivity synthesis. *10(6)*, 1968-1980.
- Prakosa, J. A., Kukaev, A. S., Parfenov, V. A., & Venediktov, V. Y. (2020). Simple automatic fluid displacement measurement by time-of-flight laser sensing technology for volume calibrator need. *Journal of Optics*, 49(1), 69-75. doi:<https://doi.org/10.1007/s12596-020-00596-5>
- Prakosa, J. A., Kurniawan, E., Sirenden, B. H., Adinanta, H., Afandi, M. I., Ula, R. K., & Pratomo, H. (2021xa). *Synthesis Method of Mixed Sensitivity for H-infinity Robust Control Optimization on DC Motor of*

- Mechatronics Model*. Paper presented at the 2021 International Conference on Radar, Antenna, Microwave, Electronics, and Telecommunications (ICRAMET), Bandung, Indonesia.
- Prakosa, J. A., Widiyatmoko, B., Bayuwati, D., & Wijonarko, S. (2021b). *Optoelectronics of Non-Contact Method to Investigate Propeller Rotation Speed Measurement of Quadrotor Helicopter*. Paper presented at the 2021 International Symposium on Electronics and Smart Devices (ISESD), Bandung, Indonesia.
- Taut, M., Chindris, G., & Pitică, D. (2018). *PID Algorithm used for DC Motor Control*. Paper presented at the 2018 IEEE 24th International Symposium for Design and Technology in Electronic Packaging (SIITME), Iasi, Romania.
- Tayal, S., Rao, H. P. G., Bhardwaj, S., & Aggarwal, H. (2020). *Line Follower Robot: Design and Hardware Application*. Paper presented at the 2020 8th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), Noida, India.
- Umar, A. (2017). Development of a Position and Trajectory Tracking Control of Ball and Plate System using a Double Feedback Loop Structure. (*Masters of Science Thesis*), Department of Electrical and Computer Engineering, Faculty of Engineering, Ahmadu Bello University, Zaria, Published.
- Umar, A., Muazu, M. B., Aliyu, U. D., Musa, U., Haruna, Z., & Oyibo, P. O. (2018). Position and Trajectory Tracking Control for the Ball and Plate System using Mixed H_{∞} Sensitivity Problem. *Covenant Journal of Informatics and Communication Technology*, (CJICT), 6(1), 1-15.
- Vinida, K., & Chacko, M. (2016). *A novel strategy using H infinity theory with optimum weight selection for the robust control of sensorless brushless DC motor*. Paper presented at the 2016 IEEE symposium on sensorless control for electrical drives (SLED), Nadi, Fiji.
- Vinida, K., & Chacko, M. (2021). Implementation of Speed Control of Sensorless Brushless DC Motor Drive using H-infinity Controller with Optimized Weight Filters. *International Journal of Power Electronics Drive Systems*, 12(3), 1379. doi:10.11591/ijpeds.v12.i3.pp1379-1389
- Xue, D., Chen, Y., & Atherton, D. P. (2007). *Linear Feedback Control; Analysis and Design with MATLAB* (Vol. 3). 3600 Market Street, 6th floor, Philadelphia, PA 19104-2688, USA: Society for Industrial and Applied Mathematics.